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COMPARATIVE STUDY ON END MOMENTS REGARDING APPLICATION OF SLOPE DEFLECTION, FLEXIBILITY MATRIX METHOD AND STAAD.Pro SOFTWARE

A.Udaykumar*
M.Chiranjeevi**
S.V.Saikumar***
M. Mohan****

Abstract

In the analysis of continuous beam, there are many complexities and monotonous calculations involved in traditional methods. End moment is one of the important parameter from the structure design point of view. In these present investigations to calculate end moment's two different methods i.e. Slope deflection and flexibility matrix method have been applied for the analyses of the continuous beam. The study reveals that the End moments obtained from these methods have nearly same value when compared with STAAD.Pro Software. The results obtained from slope deflectio method are quite accurate value then the compare with the STAAD.Pro.

Keywords:

Continuous Beam;
Slope Deflection Method;
Flexibility Matrix Method;
STAAD.Pro.

Author correspondence:

A.Uday Kumar,
Asst.Prof, Dept.of Civil Engg.
Bits Vizag, P.M Palem, Madhurawada, Visakhapatnam.

1. Introduction

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy the equilibrium equations (implying that the structure is in equilibrium) compatibility equations (requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

* Assistant Professor, Dept.of Civil Engg, Bits Vizag, Madhurawada, Visakhapatnam.
** Assistant Professor, Dept.of Civil Engg, Bits Vizag, Madhurawada, Visakhapatnam.
*** Assistant Professor, Dept.of Civil Engg, Bits Vizag, Madhurawada, Visakhapatnam.
**** Assistant Professor, Dept.of Civil Engg, Bits Vizag, Madhurawada, Visakhapatnam.

1. Force method of analysis
2. Displacement method of analysis

In the force method of analysis, primary unknown are forces.In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium.

In the displacement method of analysis,the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

2. METHODS FOR STRUCTURAL ANALYSIS

Following two methods used for the analysis.

- Slope deflection method.
- Flexibility Matrix method.

2.1 SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.

The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames. In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known. The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

2.2 FLEXIBILITY MATRIX METHOD

The force method was originally developed by J.G. Maxwell and refined by Otto Mohr and Muller-Breslau. This method was one of the first available for the analysis of statically indeterminate structures. The force method consists of writing equations that satisfy the compatibility and forces displacement requirements for the structure and involve redundant forces as the unknowns. The coefficients of these unknowns are called flexibility coefficients. Since compatibility forms the basis for this method, it has sometimes been referred to as the compatibility method or the method of consistent deformations. Once the redundant forces have been determined, the structure is determined by satisfying the equilibrium requirements for the structure. The fundamental principles involved in applying this method are easy to understand and develop.

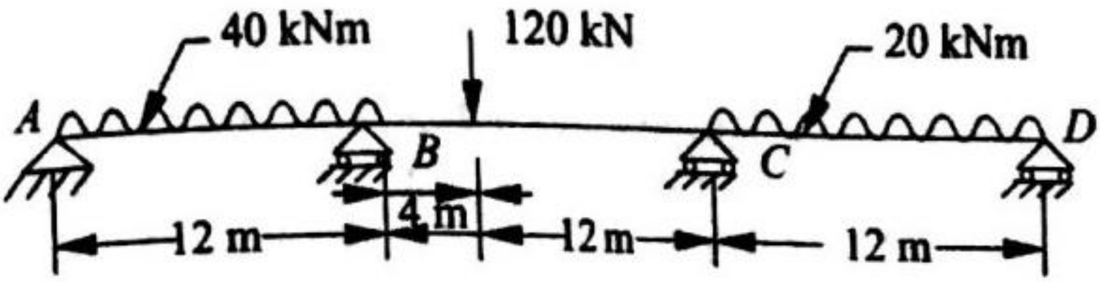
The systematic development of consist deformation method in the matrix from has lead to flexibility matrix method. The method is also called force method. Since the basic unknowns are the redundant forces in the structure. This method is exactly opposite to stiffness matrix method.

2.3 STAAD.Pro SOFTWARE

In every aspect of human civilization we needed structures to live in or to get what we need. But it is not only building structures but to build efficient structures so that it can fulfill the main purpose for what it was made for. Here comes the role of civil engineering and more precisely the role of analysis of structure. There are many classical methods to solve design problem, and with time new software's also coming into play. Here in this project work based on software named staad pro has been used. Few standard problems also have been solved to show how staad pro can be used in different cases. These typical problems have been solved using basic concept of loading, analysis, condition as per IS code. These basic techniques may be found useful for further analysis of problems.

3. METHODLOGY

This paper presents the analysis of the continuous beam shown in figure below, which is the most common in practice by using two most common methods via flexibility matrix method & slope deflection method. The moment of inertia of the continuous beam is taken as I.



3.1 ANALYSIS OF BEAM USING SLOPE DEFLECTION METHOD

3.1.1 Determination of Fixed End Moments

Let us assume that given beam has fixed end

$M_{FAB} = -\frac{WL^2}{12} = -\frac{40*(12)^2}{12} = -480$ kN-m.

$M_{FAB} = \frac{WL^2}{12} = \frac{40*(12)^2}{12} = 480$kN-m.

$M_{FBC} = -\frac{Wab^2}{12} = -\frac{120*4*(8)^2}{12^2} = -213.33$ KN-m.

$M_{FCB} = -\frac{Wa^2b}{12} = -\frac{120*(4)^2 8}{12^2} = 106.67$ kN-m.

$M_{FCD} = -\frac{WL^2}{12} = -\frac{20*(12)^2}{12} = -240$ kN-m .

$M_{FDC} = \frac{WL^2}{12} = \frac{20*(12)^2}{12} = 240$ kN-m.

3.1.2 Slope Delection equation of Given Continuous Beams

The slope deflection equations in terms of unknowns are

$M_{AB} = M_{FAB} + \frac{2EI}{L_{AB}}(2\theta_A + \theta_B) = -480 + \frac{2EI}{12}(2\theta_A + \theta_B) = -480 + \frac{EI}{6}(2\theta_A + \theta_B).$

$M_{BA} = M_{FBA} + \frac{2EI}{L_{BA}}(2\theta_B + \theta_A) = 480 + \frac{EI}{6}(2\theta_B + \theta_A).$

$M_{BC} = M_{FBC} + \frac{2EI}{L_{BC}}(2\theta_B + \theta_A) = -213.33 + \frac{EI}{6}(2\theta_B + \theta_C).$

$M_{CB} = M_{FCB} + \frac{2EI}{L_{CB}}(2\theta_C + \theta_B) = 106.67 + \frac{EI}{6}(2\theta_C + \theta_B).$

$M_{CD} = M_{FCD} + \frac{2EI}{L_{CD}}(2\theta_C + \theta_D) = -240 + \frac{EI}{6}(2\theta_C + \theta_D).$

$M_{DC} = M_{FDC} + \frac{2EI}{L_{DC}}(2\theta_D + \theta_C) = 240 + \frac{EI}{6}(2\theta_D + \theta_C).$

3.1.3 Applying Equilibrium Conditions to Slope Deflection Equations

The end A is simply supported, $M_{AB} = 0$

$$-480 + \frac{EI}{6}(2\theta_A + \theta_B) = 0$$

$$\frac{EI}{6}(2\theta_A + \theta_B) = 480$$

$$(2\theta_A + \theta_B) = \frac{2880}{EI}$$

$$2\theta_A = \frac{2880}{EI} - \theta_B$$

$$\theta_A = \frac{1440}{EI} - \frac{\theta_B}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)\ eq.$$

The end 'D' is simply supported hence $M_{DC} = 0$

$$240 + \frac{EI}{6}(2\theta_D + \theta_C) = 0$$

$$(2\theta_D + \theta_C) = \frac{-240*6}{EI}$$
$$(2\theta_D + \theta_C) = \frac{-1440}{EI}$$
$$\theta_D = \frac{-720}{EI} - \frac{\theta_C}{2} \quad \text{--- (2) eq.}$$

For equilibrium the sum of moments at support 'B' is zero

$M_{BA} + M_{BC} = 0$

$$480 + \frac{EI}{6}\left(2\theta_B + \frac{1440}{EI} - \frac{\theta_B}{2}\right) - 213.33 + \frac{EI}{6}(2\theta_B + \theta_C) = 0$$
$$266.67 + \frac{EI}{6}\left(2\theta_B + \frac{1440}{EI} - \frac{\theta_B}{2} + 2\theta_B + \theta_C\right) = 0$$
$$266.67 + \frac{EI}{6}\left(4\theta_B + \frac{1440}{EI} - \frac{\theta_B}{2} + \theta_C\right) = 0$$
$$266.67 + \frac{EI}{6}\left(4\theta_B - \frac{\theta_B}{2} + \theta_C\right) + \frac{1440}{6} = 0$$
$$\frac{EI}{6}\left(4\theta_B - \frac{\theta_B}{2} + \theta_C\right) = -506.67$$
$$\left(4\theta_B - \frac{\theta_B}{2} + \theta_C\right) = -\frac{3040.02}{EI}$$
$$(8\theta_B - \theta_B + 2\theta_C) = -\frac{3040.02*2}{EI}$$
$$(7\theta_B + 2\theta_C) = \frac{6080.04}{EI} \quad \text{--- (3) eq.}$$

For equilibrium the sum of moments at joint 'c' is zero

$M_{CB} + M_{CD} = 0$

$$106.67 + \frac{EI}{6}(2\theta_C + \theta_B) - 240 + \frac{EI}{6}(2\theta_C + \theta_D) = 0$$
$$106.67 + \frac{EI}{6}(2\theta_C + \theta_B) - 240 + \frac{EI}{6}\left(2\theta_C - \frac{720}{EI} - \frac{\theta_C}{2}\right) = 0$$
$$106.67 + \frac{EI}{6}\left(2\theta_C + \theta_B + 2\theta_C - \frac{\theta_C}{2}\right) - 120 - 240 = 0$$
$$\left(2\theta_C + \theta_B + 2\theta_C - \frac{\theta_C}{2}\right) = \frac{1519.98}{EI}$$
$$\left(4\theta_C + \theta_B - \frac{\theta_C}{2}\right) = \frac{1519.98}{EI}$$
$$(7\theta_C + 2\theta_B) = \frac{3039.96}{EI} \quad \text{--- (4) eq.}$$

Eq. (4) multiply by 7 we get

$$(49\theta_C + 14\theta_B) = \frac{21279.72}{EI} \quad \text{--- (5) eq.}$$

Eq. (3) multiply by 2 we get

$$(4\theta_C + 14\theta_B) = -\frac{12160.08}{EI} \quad \text{--- (6) eq.}$$

From (5) – (6) we get

$$(49\theta_C + 14\theta_B - 4\theta_C - 14\theta_B) = \frac{21279.72}{EI} + \frac{12160.08}{EI}$$
$$45\theta_C = \frac{33439.8}{EI}$$
$$\theta_C = \frac{743.10}{EI}.$$

From eq. (4)

$$2\theta_B = \frac{3039.96}{EI} - \left(\frac{743.10}{EI}\right)*7$$
$$\theta_B = -\frac{1080.87}{EI}$$

From eq. (1)

$$\theta_A = \frac{1980.435}{EI}$$

From eq. (2)

$$\theta_D = -\frac{720}{EI} - \frac{743.10}{2EI}$$
$$\theta_D = -\frac{1091.55}{2EI}.$$

3.1.4 FINAL MOMENTS

$M_{AB} = 0$

$M_{BA} = -480 + \frac{EI}{6}\left[2\left[\frac{-1080.87}{EI}\right] + \frac{1980.435}{2EI}\right] = 450$ KN-m.

$M_{BC} = -213.33 + \frac{EI}{6}\left[2\left[\frac{-1080.87}{EI}\right] + \frac{743.10}{2EI}\right] = -450$ KN-m.

$M_{CB} = 106.67 + \frac{EI}{6}\left[2\left[\frac{743.10}{EI}\right] - \frac{1080.87}{2EI}\right] = 174.225$ KN-m.

$M_{CD} = -240 + \frac{EI}{6}\left[2\left[\frac{743.10}{EI}\right] - \frac{1091.55}{2EI}\right] = -174.225$KN-m.

$M_{DC} = 0$

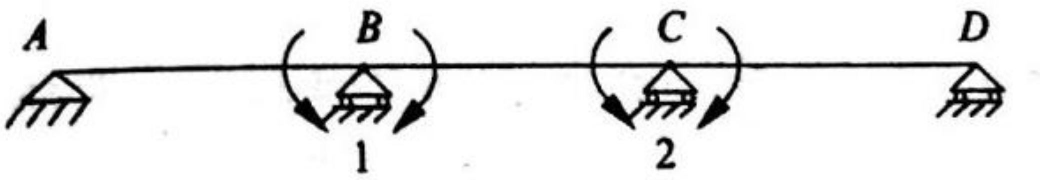
3.2 ANALYSIS OF BEAM USING FLEXIBILITY MATRIX METHOD

3.2.1 Determination of Static Indermenancy

Number of reaction components = 2 + 1 + 1 + 1 = 5.
Number of independent equilibrium equations = 3.
Degree of static indetermenancy n = 5- 3 = 2

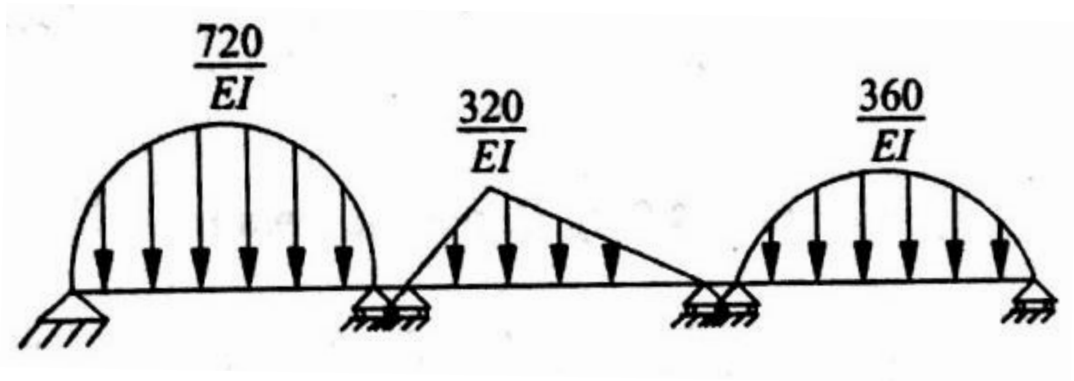
3.2.2 Assign the co-ordinates and draw released beam diagram

The moments at B and C are taken as redundant forces. Hence, released structure is obtained by introducing the hinges at B and C.the coordinate chosen are as shown in figure below.



3.2.3 Conjugate beam diagram

Displacements due to given loadings in the released structure: the conjugate beam and the loads on it are as shown in figure below



The maximum ordinate of load in AB = $\frac{1}{EI} * 40 * \frac{12^2}{8} = \frac{720}{EI}$
The maximum ordinate of load in BC = $\frac{1}{EI} * \frac{120*4*8}{8} = \frac{320}{EI}$
The maximum ordinate of load in CD = $\frac{1}{EI} * 20 * \frac{12^2}{8} = \frac{360}{EI}$

3.2.4 Determination of Displacement Matrix

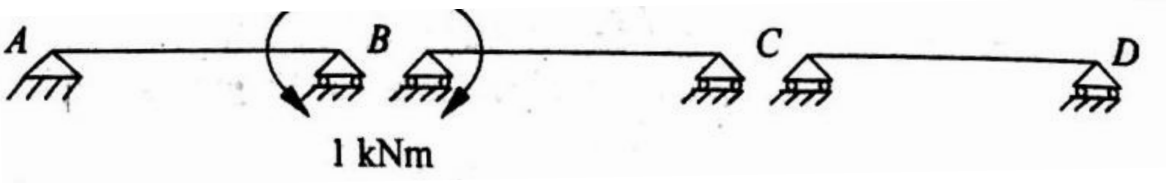
Δ_{1L}= Displacement in coordinate direction 1
= Rotation of AB at B + Rotation of BC at B
= Shear force in conjugate beam in BC at B + Shear force in conjugate beam in BC at B.

$$= \frac{1}{2} * \frac{2}{3} * \frac{720}{EI}*12+\frac{1}{2} * \frac{320}{EI}*12*\frac{(12+8)}{3}*\frac{1}{12} = \frac{3946.67}{EI}$$

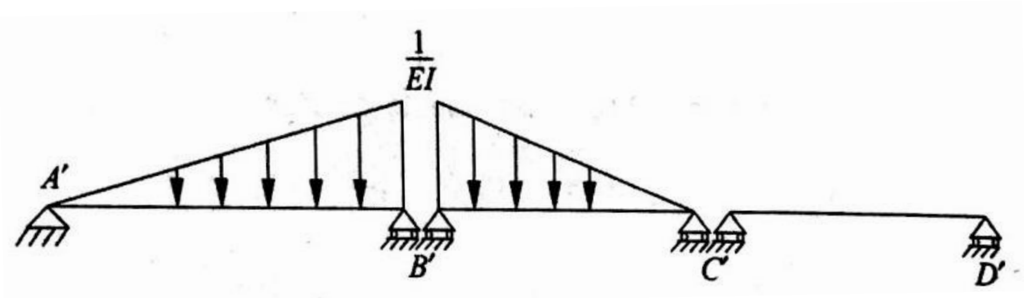
$$\Delta_{2L} = \frac{1}{2} * \frac{320}{EI} * 12 * \frac{(12+4)}{3}*\frac{1}{12} + \frac{1}{2}\left(\frac{2}{3}\right) * \frac{360}{EI}*12 = \frac{2293.33}{EI}$$

3.2.5 Determination of Flexibility Matrix

(a) Unit force is applied in determinate structure in the coordinate direction 1 as shown in fig. Conjugate beams and loading on them are as shown in fig.



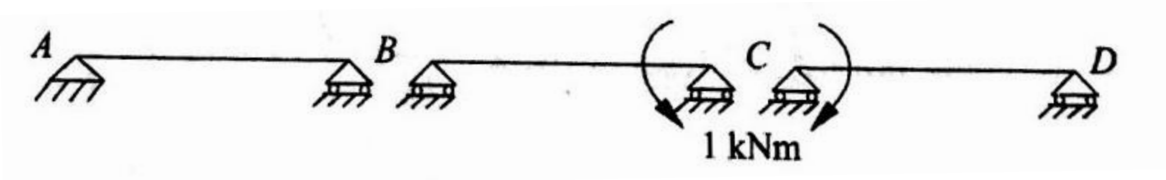
Released structure with unit load in coordinate direction 1.



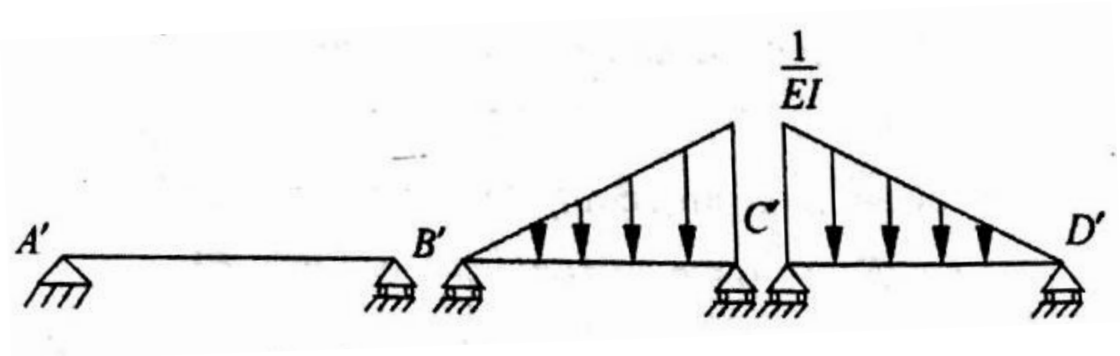
Conjugate Beam

$$\delta_{11} = \frac{2}{3} * \frac{1}{2} * 12 * \frac{1}{EI} + \frac{2}{3} * \frac{1}{2} * 12 * \frac{1}{EI} = \frac{8}{EI}$$
$$\delta_{21} = \frac{1}{3} * \frac{1}{2} * 12 * \frac{1}{EI} = \frac{2}{EI}$$

(b)Unit force is applied in coordinate direction 2 as shown in fig. The conjugate beam and loading on it are as shown in fig.



Released structure with unit load in coordinate direction 2.



Conjugate Beam

$$\delta_{21} = \frac{1}{3} * \frac{1}{2} * 12 * \frac{1}{EI} = \frac{2}{EI}$$
$$\delta_{22} = \frac{2}{3} * \frac{1}{2} * 12 * \frac{1}{EI} + \frac{2}{3} * \frac{1}{2} * 12 * \frac{1}{EI} = \frac{8}{EI}$$

The final displacement at 1 and 2 are zero. Hence, the flexibility equation is given by

$$[\delta][P] = [\Delta] - [\Delta_L]$$

$$\begin{bmatrix} \frac{8}{EI} & \frac{2}{EI} \\ \frac{2}{EI} & \frac{8}{EI} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{3946.67}{EI} \\ \frac{2293.33}{EI} \end{bmatrix}$$

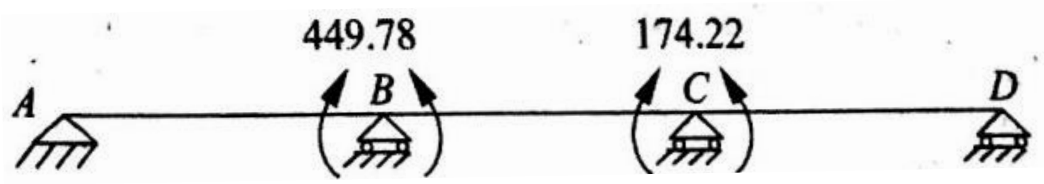
$$\frac{1}{EI}\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\frac{1}{EI}\begin{bmatrix} 3946.67 \\ 2293.3 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 3946.67 \\ 2293.3 \end{bmatrix}$$

$$= \frac{1}{(64-4)}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} 3946.67 \\ 2293.3 \end{bmatrix}$$

$$= \begin{bmatrix} -449.78 \\ -174.22 \end{bmatrix}$$

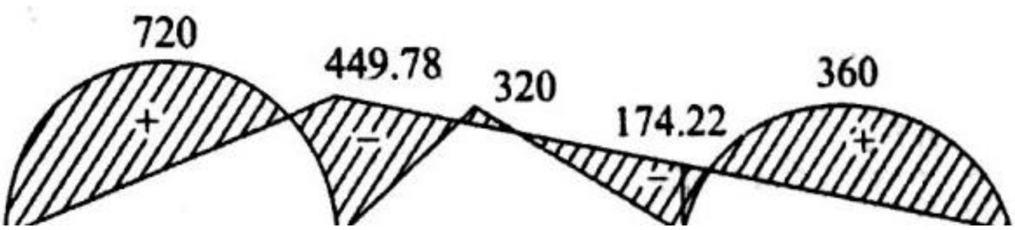
The end moments obtained are indicated in fig.



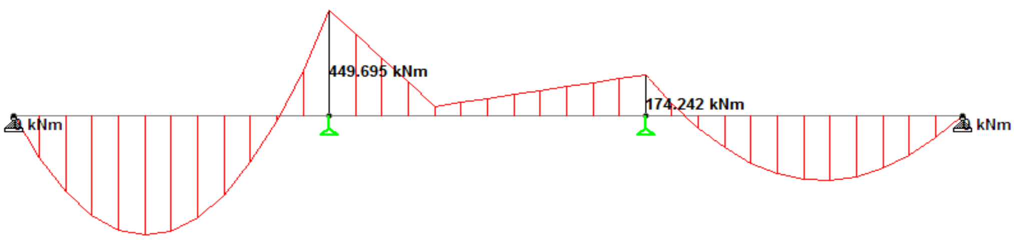
3.2.6 Final Moments

$M_{AB} = 0$
$M_{BA} = -449.78$ KN-m.
$M_{BC} = 449.78$KN-m.
$M_{CB} = -174.22$ KN-m.
$M_{CD} = 174.22$ KN-m.
$M_{DC} = 0$

Hence, the ending moment diagram is as shown in fig.



3.3 ANALYSIS OF BEAM USING STAAD.Pro SOFTWARE



4. RESEARCH ANALYSIS

After the analysis is completed, the results obtained from slope deflection method and Flexibility matrix method has been compared with results obtained from Staad.Pro.

Table.3: Comparison of End Moments by Manual Methods and STAAD.Pro

Moment at	**End Moments (kN-m)**	**STAAD.Pro Software**

	Analytical Methods		
	Slope Deflection Method	**Flexibility Matrix Method**	**Staad.Pro**
M_{AB}	**0**	**0**	**0**
M_{BA}	**450**	**449.78**	**449.65**
M_{BC}	**-450**	**-449.78**	**-449.65**
M_{CB}	**174.225**	**174.225**	**174.242**
M_{CD}	**-174.225**	**-174.225**	**-174.242**
M_{DC}	**0**	**0**	**0**

5. Conclusion

The End Moments of a Continuous beam solved by the application of Slope Deflection Method and Flexibility Matrix Method successfully substanntiated by using STAAD.Pro software. In this paper the results obtained from manually by two different methods are comparing with the results obtained from STADD.Pro software. There is slightly variation in the value of End Moments by manual analysis as well as software analysis. In manual calculation the Flexibility Matrix Method results are nearly same when compared with STAAD.Pro results.

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